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Structure-Aligned Geometric Compensation for Additive Manufacturing Warpage via Vibrational Modal Analysis

Additively manufactured parts can deviate from their nominal design due to process-induced deformation, such as residual-stress-driven warpage during thermal cycling, reducing dimensional accuracy. Geometric compensation seeks to correct this deviation by predeforming the design so that the printed part recovers its intended shape. Most existing approaches construct compensation by directly reversing the measured deformation field. However, direct inversion can propagate nonstructural components into the compensation rather than isolating the main structural deformation. To address this limitation, we reformulate geometric compensation as a representation problem of deformation, rather than purely an inversion problem. Accordingly, we reformulate compensation within a reduced structural modal subspace derived from vibrational modal analysis of the nominal structure. This modal representation restricts compensation to deformation patterns consistent with the part's structural response. Finite-element simulations show that the proposed formulation effectively compensates additive-manufacturing warpage when deformation is dominated by structural response, remains robust to measurement and reconstruction errors, and suggests broader applicability to other geometry correction problems driven by coherent structural deformation. [DOI: 10.1115/1.4072276]

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1 Introduction

Geometric deformation is a persistent challenge in additive manufacturing, where process-induced deformation, such as residual-stress-induced warpage, reduces dimensional accuracy and process reliability [1]. Geometric compensation mitigates this issue by predeforming the nominal design so that the printed part recovers its intended geometry. Most compensation methods operate directly on the observed deformation field for efficiency and simplicity [2]. They are typically formulated as a direct inversion [3–5], where the deviation between the as-built and nominal shapes is represented as a displacement field.

However, existing geometric compensation based on direct inversion lacks an explicit structural constraint and treats deformation as a full geometric field. This treatment does not distinguish the underlying structural response from nonstructural components such as noise, registration errors, or correspondence artifacts. In additive manufacturing, the observed deformation is not an arbitrary geometric offset; it is governed by spngness distribution, boundary conditions, and process-induced loading and, therefore, often appears as spatially coherent structural deformation.

When the structural nature of a part is ignored, nonstructural components such as correspondence errors, registration artifacts, and measurement noise may be incorporated into the deformation field and propagated into the compensation field, leading to local overcorrection and less stable compensation [6,7]. Geometric smoothing and numerical regularization are often used to mitigate these effects [8,9], but they do not resolve the mismatch between

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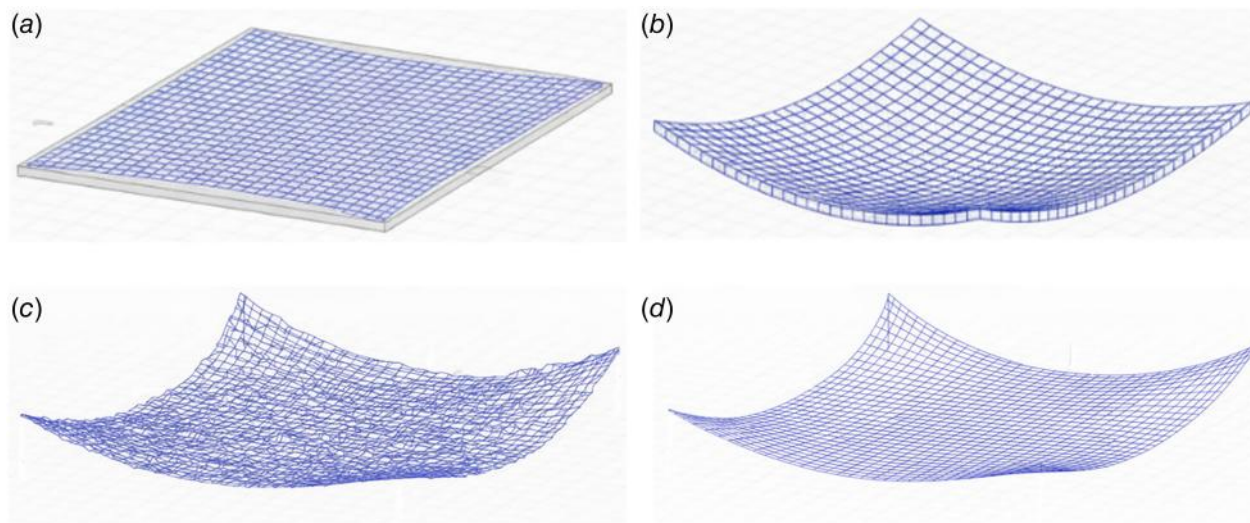


Fig. 1 Comparison of full-field and reduced mode-based deformation representations for the same warped plate: (a) Nominal design geometry, (b) Built geometry, (c) Full-field measured geometric deformation, and (d) Reduced modal representation of deformation.

defining compensation in the full geometric field and the structure-governed nature of the deformation [10–13].

Figure 1 illustrates the difference between representing deformation directly in the full geometric field and representing it using a reduced set of structural deformation modes. These modes are deformation patterns associated with the structural response of the nominal part. The full-field representation retains all observed components of the deformation, including localized noise, correspondence artifacts, and other nonstructural variations. In contrast, the reduced mode-based representation preserves the dominant structural patterns while suppressing localized components that are inconsistent with the global structural behavior.

The central issue in geometric compensation is therefore not only the inversion of a measured deformation field, but also the representation space where compensation is constructed. To define this structure-aligned space, we use vibrational modal analysis [14]. In this analysis, the eigenmodes of the nominal structure describe its intrinsic displacement patterns and are ordered by increasing natural frequency. These modes provide a basis for representing admissible structural deformation. When process-induced deformation is dominated by global bending or warpage, its main spatial content is expected to be captured by a small number of low-frequency modes. By retaining only these modes, compensation is constructed within coherent structural deformation patterns rather than the full geometric displacement field.

We therefore reformulate geometric compensation as a problem of deformation representation rather than geometric inversion. We propose a structure-aligned modal compensation formulation, in which compensation is constructed within a reduced modal subspace defined by the dominant structural eigenmodes.

This work provides three contributions. First, it shows that geometric compensation is fundamentally a representation problem rather than only an inversion problem, motivating the need for a structure-aligned compensation space. Second, it introduces a structure-aligned modal compensation formulation based on the structural eigenmodes of the nominal structure. By constructing compensation within a low-dimensional modal subspace, the method confines the correction to dominant structural deformation patterns and reduces the influence of correspondence errors, registration artifacts, measurement noise, and reconstruction errors. Third, it evaluates the proposed formulation through finite-element numerical studies, including a plate benchmark, an L-bracket geometry, closed-loop compensation validation, and perturbation analyses. Compared with direct inversion, the proposed formulation achieves lower residual deformation in the tested closed-loop

validations, produces smoother and more structurally coherent compensation fields, and improves robustness under measurement and correspondence perturbations. More broadly, the work establishes a representation-based framework for geometric compensation and suggests a physically interpretable path toward robust deformation correction under imperfect observations.

The remainder of this article is organized as follows. Section 2 reviews direct inversion-based compensation approaches and their representation limitations. Section 3 presents the proposed modal formulation. Section 4 evaluates the method through benchmark studies, closed-loop validation, and perturbation analysis. Section 5 discusses applicability, limitations, and future extensions. Conclusions are given in Sec. 6.

2 Related Work

Prior work on geometric compensation can be reviewed according to how the deformation field is obtained or estimated: process simulation, data-driven prediction, or measurement-based identification. Although these approaches differ in how they generate the deformation field, the subsequent compensation step is generally defined as the inversion of a full-field geometric displacement. This section reviews these direct inversion-based approaches and then discusses their representation limitations and empirical remedies.

2.1 Direct-Inversion-Based Compensation. Early and representative realizations of geometric inversion formulation were developed in the context of springback compensation in metal forming. In these approaches, full-field deformation obtained from finite-element simulations of the forming and unloading processes was inverted to iteratively update the tooling or die geometry [3,15]. Subsequent work adopted the same geometric inversion principle, while refining the numerical procedure through iterative updates or relaxation strategies to improve convergence [4].

Within the direct inversion framework, many studies obtain the deformation field from process-level thermo-mechanical simulations of the manufacturing process. The predicted full-field nodal displacement is subsequently inverted to construct compensation, while individual models primarily differ in their treatment of residual stress, thermal loading, and process physics across manufacturing contexts such as additive manufacturing and machining [5,16–18].

Data-driven prediction approaches infer deformation from process-related or simulation-generated data without explicitly

modeling the underlying process physics. Representative methods include neural networks trained to learn deformation responses from process parameters or simulation outputs [19,20], as well as statistical formulations such as Gaussian process-based surrogates with Bayesian calibration [21], probabilistic regression models for directly inferring geometric deviations [22], and self-consistent clustering approaches for multiscale multiphysics modeling in additive manufacturing [23].

Separately, deformation fields can be identified directly from measurements of fabricated parts [24]. In these measurement-based approaches, scan-based deformation identification is formulated as a statistical inference problem that jointly resolves correspondence and displacement [25] and is further embedded within statistical modeling frameworks [26]. These studies extend geometric compensation to scan-based AM workflows, but the correction is still constructed from the identified full-field displacement.

2.2 Representation Limitations and Empirical Remedies.

A recurring limitation of direct-inversion-based geometric compensation is its sensitivity to unstable geometric updates. When geometric compensation is implemented through repeated direct inversion, nonconvergent behavior has been reported across a range of manufacturing processes and deformation cases. In particular, repeated reversal of the estimated displacement field does not guarantee monotonic error reduction and has been observed to produce oscillatory updates or divergence from the target geometry, even when the underlying deformation prediction is accurate [9,27].

To mitigate these numerical instabilities, a variety of procedural adjustments have been adopted in practice. Common strategies include performing compensation in a single-shot or weakly iterative manner to avoid repeated geometry updates [4,5], as well as limiting the update magnitude through relaxation or scaling factors [27]. These measures regulate how inversion is applied across iterations but do not change the underlying geometric formulation or address the representation-level mismatch between structurally governed deformation and unconstrained geometric correction.

Other approaches modify the estimated deformation field prior to inversion in order to suppress unstable geometric components. Instead of applying full-field displacement reversal, the deformation is smoothed or restricted to selected features, such as global shrinkage or spring-in [9,28]. A further development avoids direct pointwise inversion by introducing a parameterized representation of geometric correction. A reduced set of geometric control variables is then iteratively adjusted to minimize deviation from the target shape [29].

Taken together, prior studies on geometric compensation employ a range of modeling, prediction, and measurement techniques to obtain deformation fields, but the subsequent correction step generally relies on geometric displacement inversion. Although smoothing, regularization, and data-driven prediction can improve robustness in practice, these developments largely retain the same full-field formulation for defining compensation.

Vibrational modal analysis offers a different representation perspective. It has long been used in structural dynamics to represent structural response through modal superposition [14] and in component mode synthesis to construct reduced-order dynamic models [30]. Beyond structural dynamics, modal representations have also been used as compact deformation spaces in computer graphics and geometry processing, including modal dynamics for graphics and animation [31], constrained interactive deformation [32], and low-frequency modal subspaces for interactive surface modeling [33]. These studies show that modal bases can provide compact and physically meaningful deformation spaces. This suggests a different way to formulate compensation: rather than directly inverting the full observed deformation field, the correction can be constructed in a structure-aligned space associated with the structural response. Accordingly, we propose a structure-

aligned modal compensation formulation that represents and constructs the compensation field within a reduced modal subspace defined by the dominant structural eigenmodes of the nominal part.

3 Methodology: Modal-Based Compensation

We formulate geometric compensation in a reduced modal representation derived from the nominal structure, rather than directly in the full geometric displacement field. The observed deformation is first expressed on a common reference configuration by registering the deformed geometry and establishing correspondence with the nominal geometry. It is then projected onto the retained structural eigenmodes to identify modal coefficients, and compensation is constructed within this reduced modal subspace.

Figure 2 summarizes this workflow: registration and correspondence establish the observed deformation field, modal projection represents it in the reduced structural subspace, and the resulting modal compensation generates a predeformed design for compensated manufacturing.

3.1 Compensation Problem Formulation. Let the nominal as-designed part be represented in a discrete form suitable for numerical analysis, such as a finite element mesh derived from a CAD or STL model. In this work, the geometry is represented by a finite element mesh with N nodes. The stacked nodal coordinate vector of the nominal configuration is denoted by

$$\mathbf{q}^{\text{nom}} = [\mathbf{q}_1^T \quad \mathbf{q}_2^T \quad \cdots \quad \mathbf{q}_N^T]^T \in \mathbb{R}^{3N}$$

where $\mathbf{q}_l \in \mathbb{R}^3$ denotes the coordinate vector of node l in the nominal geometry, $l = 1, \dots, N$. After manufacturing or process simulation, the part assumes a deformed geometry represented by the stacked nodal coordinate vector

$$\mathbf{q}^{\text{out}} \in \mathbb{R}^{3N}$$

expressed on the same reference configuration and nodal ordering as the nominal geometry.

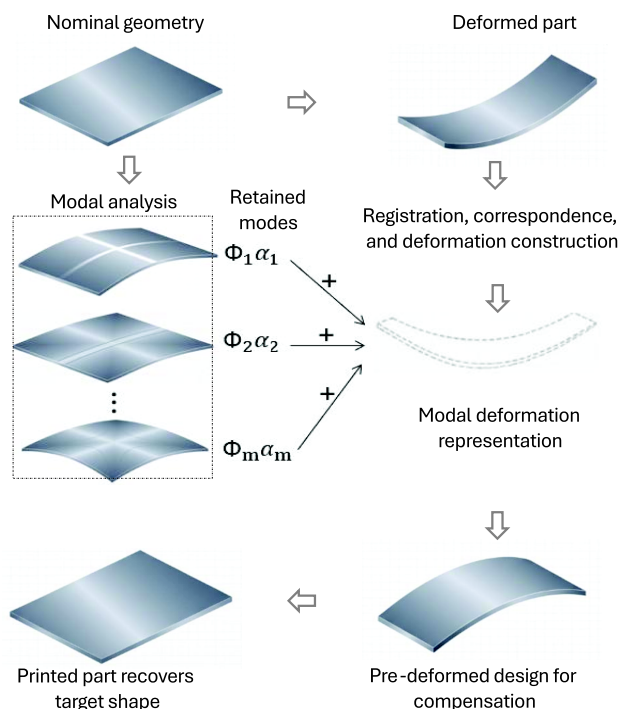


Fig. 2 Conceptual illustration of the proposed modal-based geometric compensation workflow

The deformed geometry is typically obtained from 3D optical scanning of the as-built part or from process simulation, and is registered to the nominal geometry before deformation extraction. Establishing geometric correspondence is treated as a preprocessing step and is not considered further in this work. The manufacturing-induced geometric deviation is represented as a nodal displacement field defined on the nominal geometry:

$$\mathbf{D}_{\text{obs}} = \mathbf{q}^{\text{out}} - \mathbf{q}^{\text{nom}} \in \mathbb{R}^{3N} \quad (1)$$

where the subscript obs denotes the observed deformation.

Given the observed deformation field $\mathbf{D}_{\text{obs}}$, geometric compensation seeks to define a compensation displacement field $\mathbf{C} \in \mathbb{R}^{3N}$ that modifies the nominal design geometry.

In existing direct inversion-based formulations, the compensation field is defined directly in the geometric displacement space by prescribing

$$\mathbf{C}_{\text{DI}} = -\mathbf{D}_{\text{obs}} \quad (2)$$

thereby expressing compensation explicitly as a function of the observed geometric deformation. This may transfer high-frequency or nonstructural components in the observed field into the compensated surface.

In contrast, the proposed approach defines the compensation field

$$\mathbf{C}_{\text{modal}} \in \mathbb{R}^{3N} \quad (3)$$

through a structure-aligned modal representation of deformation. Rather than inverting the observed deformation in the geometric displacement space, $\mathbf{C}_{\text{modal}}$ is constructed by reformulating the compensation problem in a modal space aligned with the dominant structural response, as detailed in the following subsection.

Once a compensation displacement field $\mathbf{C}$ is defined, the compensated design geometry is obtained by applying the displacement to the nominal design:

$$\mathbf{q}^{\text{comp}} = \mathbf{q}^{\text{nom}} + \mathbf{C} \quad (4)$$

In the numerical evaluation, closed-loop validation is performed by using $\mathbf{q}^{\text{comp}}$ as the new input geometry and reapplying the same deformation process to evaluate the residual deviation from the nominal target.

3.2 Structure-Aligned Modal Representation. Deformation is represented in a modal space derived from the nominal configuration. The structural eigenmodes, determined by the nominal spngness and mass properties, span the linearized deformation space of the structure. When these properties remain representative, the modal basis provides a physically meaningful description of structural deformation [14].

For the finite element discretization of the nominal geometry, let $\mathbf{K} \in \mathbb{R}^{3N \times 3N}$ and $\mathbf{M} \in \mathbb{R}^{3N \times 3N}$ denote the spngness and mass matrices, respectively. Under linear elasticity, vibration modes are obtained from the generalized eigenvalue problem [14]:

$$\mathbf{K}\Phi = \mathbf{M}\Phi\Lambda \quad (5)$$

where the columns of $\Phi = [\phi_1, \phi_2, \dots]$ are eigenvectors and $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots)$ contains the corresponding eigenvalues in ascending order.

Eigenvectors associated with zero or near-zero eigenvalues correspond to rigid-body motions and are excluded, since they do not represent shape change. The remaining shape-changing eigenvectors are mass-normalized such that

$$\Phi^T \mathbf{M} \Phi = \mathbf{I} \quad (6)$$

and are used to construct the reduced deformation subspace for compensation.

In many additive-manufacturing deformation problems, the observed deformation is governed primarily by a small number

of low-frequency structural modes. The deformation can therefore often be approximated in a low-dimensional modal subspace, where a truncated representation provides a compact description using only the dominant modes.

Let $\mathbf{D} \in \mathbb{R}^{3N}$ denote a reference deformation field in vectorized nodal form, and let $\Phi_k \alpha_k$ denote its projection onto the first k retained structural modes. To quantify the fraction of deformation captured by these modes, the cumulative captured deformation energy ratio under the $\mathbf{M}$ -weighted norm is defined as

$$E_k = 1 - \frac{\|\mathbf{D} - \Phi_k \alpha_k\|_{\mathbf{M}}^2}{\|\mathbf{D}\|_{\mathbf{M}}^2} \quad (7)$$

where $\|\mathbf{x}\|_{\mathbf{M}}^2 = \mathbf{x}^T \mathbf{M} \mathbf{x}$.

The truncation level is then selected as

$$m = \min \{k : E_k \geq \eta\} \quad (8)$$

where η is a fixed threshold applied consistently across all cases. The retained modal basis is

$$\Phi_{\text{low}} = [\phi_1 \quad \phi_2 \quad \dots \quad \phi_m] \in \mathbb{R}^{3N \times m} \quad (9)$$

where ϕ_i denotes the i th retained shape-changing vibration mode. The resulting truncated representation retains the leading modal components while discarding higher-order modes with limited captured contribution, yielding a compact reduced representation of the deformation.

Within the retained modal subspace, deformation fields are represented as linear combinations of the retained modes. To construct the compensation field, the observed deformation is projected onto the subspace to obtain the modal coefficient vector $\alpha_m \in \mathbb{R}^m$. The coefficient magnitude indicates how strongly each retained structural mode participates in the observed deformation.

The modal coefficients are obtained by projecting the observed deformation onto the truncated modal subspace under a mass-weighted norm,

$$\alpha_m = \arg \min_{\beta \in \mathbb{R}^m} \|\mathbf{M}^{1/2}(\mathbf{D}_{\text{obs}} - \Phi_{\text{low}} \beta)\|_2^2 \quad (10)$$

which yields the best approximation of $\mathbf{D}_{\text{obs}}$ within the truncated modal subspace.

The mass-weighted norm in Eq. (10) is used because structural eigenmodes obtained from the generalized eigenvalue problem can be chosen to be $\mathbf{M}$ -orthonormal [14]. Under mass normalization, the resulting coefficients have the standard interpretation of modal participation in the deformation approximation. In contrast, a purely geometric L_2 norm weights nodal displacements directly and can be more sensitive to mesh discretization, while a spngness-weighted norm emphasizes strain-energy content.

The projected deformation is reconstructed in physical space as

$$\mathbf{D}_{\text{modal}} = \Phi_{\text{low}} \alpha_m \quad (11)$$

The modal compensation field is then defined as the inverse of the projected deformation,

$$\mathbf{C}_{\text{modal}} = -\mathbf{D}_{\text{modal}} \quad (12)$$

Thus, inversion is applied only after the observed deformation has been restricted to the retained structure-aligned modal subspace.

In the special case where all shape-changing modes are retained and the modal basis spans the registered displacement space after rigid-body alignment, the projection becomes the identity operation. In the limiting case, the modal formulation reduces to direct inversion.

The formulation requires only a one-time offline modal analysis of the nominal structure, whose cost is comparable to a standard finite-element modal analysis. For a given nominal-deformed shape pair, the online step consists of projecting the observed deformation onto the truncated modal basis and constructing the

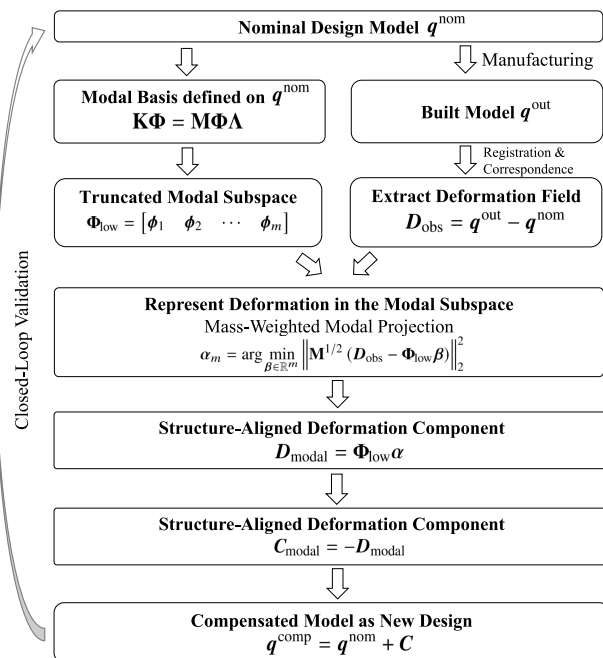


Fig. 3 Algorithmic workflow of the modal-based geometric compensation formulation

compensation field C_{modal} . The projection cost scales as $O(3Nm)$, where m is the number of retained modes, and is therefore small when $m \ll 3N$. Figure 3 summarizes the complete formulation and clarifies how geometric observation, modal representation, and compensation are connected within a unified workflow.

4 Computational Evaluation

The proposed formulation is evaluated through finite-element numerical studies, including two geometric case studies and perturbation analyses. The first case is a plate benchmark with structure-dominated warpage, and the second is an L-bracket geometry exhibiting more distributed and coupled warpage behavior. Across these studies, the analysis focuses on three aspects: modal representation of deformation, characteristics of the computed compensation field, and residual deformation after closed-loop validation. The perturbation analyses assess robustness to measurement noise, sampling sparsity, and correspondence errors. All deformation magnitudes reported in figures and tables are in millimeters (mm).

4.1 Plate Benchmark

4.1.1 Deformation Generation. A rectangular plate benchmark of dimensions $200 \text{ mm} \times 80 \times 4 \text{ mm}$ is constructed to

provide a controlled reference case for comparing direct inversion and modal-based compensation. The plate is modeled as a linear elastic solid without external constraints.

The reference deformation is generated using a finite-element model with an equivalent thermal strain formulation. A prescribed through-thickness thermal loading is applied to induce global bending, together with a mild in-plane bias to introduce controlled asymmetry. Rigid-body motion is removed numerically from the solution.

Numerical convergence with respect to in-plane mesh resolution and through-thickness discretization is verified to ensure that the reference deformation is not materially affected by numerical resolution. The deformation changes only slightly under refinement, with the normalized root-mean-square (RMS) difference below 2% when the number of thermal layers is increased from 8 to 16 and below 1% when the element size is reduced from 5 mm to 3 mm. An element size of 5 mm with eight thermal layers is therefore adopted in all subsequent analyses.

The resulting reference deformation field is shown in Fig. 4. The field varies smoothly over the plate, with the largest displacement near the free edges and a gradual reduction toward the center. The benchmark is used in the following section to examine compensation behavior under a controlled, bending-dominated deformation pattern.

4.1.2 Modal Representation. The reference deformation is projected onto the structural modal basis of the nominal geometry to examine how the deformation content is distributed across vibration modes. Rigid-body modes are excluded from the basis. Modal compactness is quantified using E_k (Eq. (7)), defined as the fraction of deformation represented by the first k retained modes under the $\mathbf{M}$ -weighted norm.

The modal distribution is shown in Fig. 5(a). The first six modes correspond to rigid-body motion and are excluded. For the plate benchmark, the first elastic mode captures approximately 94% of the deformation under the mass-weighted norm. Using the truncation criterion in Eq. (8) with $\eta = 80\%$, the required truncation level is $m = 1$, indicating that the deformation is highly compact in the modal basis. Higher modes contribute comparatively little, and the cumulative captured fraction approaches approximately 99% by mode 13. This benchmark provides a reference case of strong modal compactness, allowing the role of deformation representation to be examined under structural alignment.

Figure 5(b) shows the dominant elastic mode, which exhibits a global bending pattern consistent with the reference deformation. The limited contribution of higher-order modes indicates that only minor asymmetric components are required beyond the dominant response. This provides the structural basis for reduced-order compensation in the plate benchmark.

4.1.3 Compensation Behavior. Figure 6 compares the compensation fields obtained by direct inversion and the proposed formulation. The direct inversion solution exhibits stronger local spatial variations across the surface, whereas the proposed

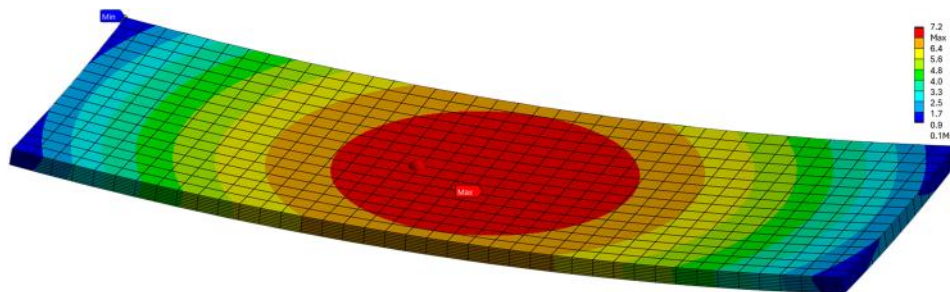


Fig. 4 Simulated reference deformation field for the plate benchmark, shown as displacement magnitude in mm

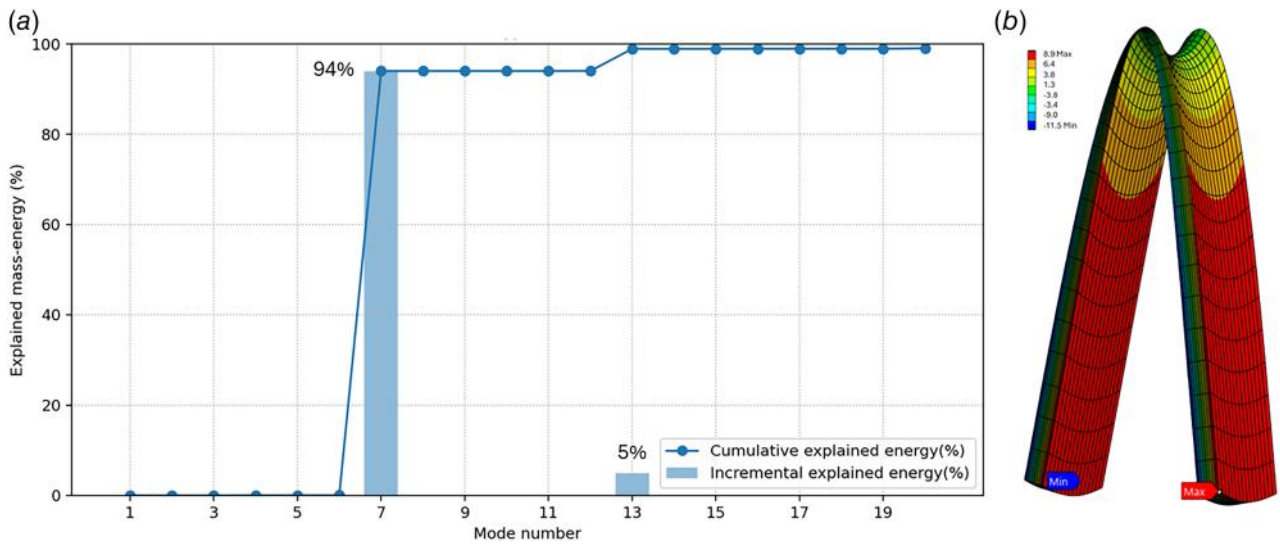


Fig. 5 Modal decomposition of the reference deformation of the plate. (a) Incremental and cumulative fraction of deformation captured by modal approximation under the mass-weighted norm. (b) Dominant elastic mode shape (mode 7).

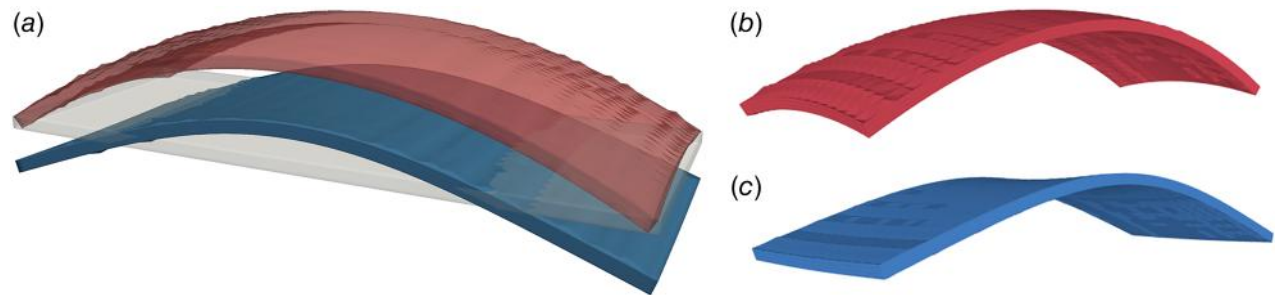


Fig. 6 Comparison of geometric compensation results for the plate. (a) Overlay of the nominal geometry (flat reference surface), direct inversion result (upper surface), and proposed modal compensation result (lower surface). (b) Direct inversion compensation. (c) Proposed modal compensation. Displacements are exaggerated by a factor of 5 for visualization.

formulation produces a smoother correction pattern aligned with the dominant plate warpage mode.

The difference is also reflected in the magnitude of the applied correction fields, summarized in Table 1. Relative to direct inversion, the proposed formulation reduces the global RMS displacement from 4.24 mm to 1.78 mm and the maximum displacement from 6.56 mm to 3.08 mm. The smaller correction magnitude reflects the removal of higher-order nonstructural components, and the effectiveness of the reduced correction is verified in the subsequent closed-loop analysis.

To compare the two compensation fields in a common structural representation, both solutions are projected onto the modal basis of the nominal geometry. Figure 7 shows that the direct inversion correction is distributed across multiple modes, including substantial higher-order content, whereas the proposed formulation remains concentrated in the leading low-order modes.

These results indicate that the observed difference between the two methods is primarily associated with deformation representation. Direct inversion retains a broader modal spectrum, whereas

the proposed formulation concentrates the correction in the dominant modes.

4.1.4 Robustness Under Perturbations. Controlled perturbations are applied to the observed deformation field of the plate benchmark. Three representative disturbance types are considered: measurement noise, spatial sparsity, and local correspondence mismatch.

Measurement noise is modeled as a 5% multiplicative Gaussian perturbation. Spatial sparsity is simulated by randomly removing 30% of nodal observations. Local correspondence mismatch is introduced by reassigning 20% of nodes to displacement values taken from one of their 20 nearest geometric neighbors. These settings represent common sources of error in scanning and reconstruction pipelines while preserving a controlled benchmark environment.

For each perturbation type, 100 independent Monte Carlo realizations are generated. For both direct inversion and the modal formulation, the compensation field computed from each perturbed observation is compared with the corresponding field obtained from the unperturbed reference deformation. RMS error quantifies the overall deviation in the correction field, while curvature error measures the change in local spatial smoothness. Smaller values indicate lower sensitivity to observation errors. Paired tests on the RMS values give $p < 10^{-3}$ for all three perturbation types.

Table 2 reports the ensemble mean of the evaluation metrics. Across all perturbation types, the modal formulation yields lower mean RMS and curvature errors than direct inversion.

Table 1 Compensation output for the plate benchmark

Metric (out-of-plane, mm)	Direct inversion	Modal (proposed)
Global RMS	4.24	1.78
Global max	6.56	3.08

Note: Bold values indicate the lower value in each comparison.

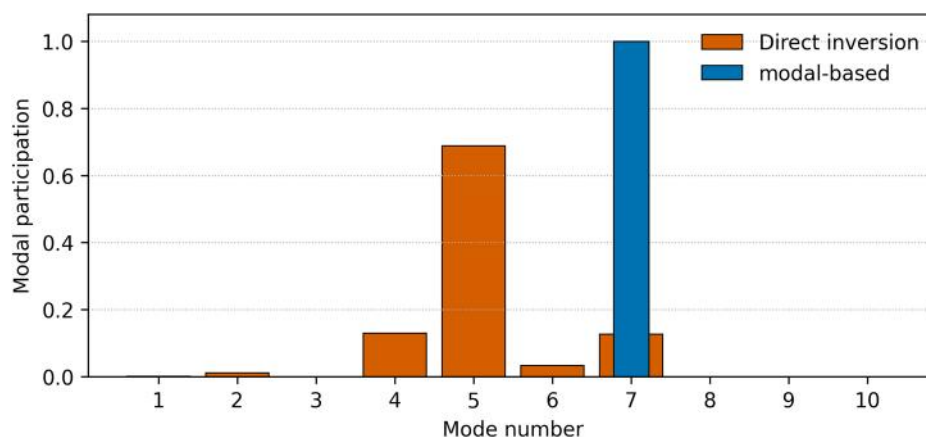


Fig. 7 Modal coefficient distributions of the direct inversion and proposed modal compensation fields for the plate. Direct inversion contains broader modal content, whereas the proposed formulation concentrates the correction in the retained dominant modes.

Table 2 Sensitivity metrics for plate benchmark compensation fields under observation perturbation

Perturbation	DI RMS	Modal RMS	DI curv.	Modal curv.
Noise	0.13	<0.01	0.17	<0.01
Sparse	1.46	0.27	1.48	0.01
Mismatch	0.03	<0.01	0.02	<0.01

Note: DI denotes direct inversion. RMS values are reported in mm. Bold values indicate the lower value in each comparison.

Under measurement noise, direct inversion exhibits an RMS displacement of 0.13, whereas the modal formulation remains below 0.01. Under spatial sparsity, direct inversion shows the largest degradation, with RMS increasing to 1.46, while the modal formulation reduces the value to 0.27. A similar contrast is observed in curvature error, where sparsity gives 1.48 for direct inversion but only 0.01 for the modal formulation. Under the local mismatch setting, the modal formulation again gives the smaller deviation.

The results suggest that the modal formulation is more robust because the tested perturbations are weakly aligned with the dominant structural modes. Restricting the correction to these modes reduces the influence of nonstructural components on the compensation field. The improved robustness therefore arises from the representation itself rather than from modifying the inversion step.

4.1.5 Closed-Loop Validation. Closed-loop performance is examined through a single-pass finite-element reloading analysis. The compensated geometry is subjected to the same equivalent thermal loading, and the remaining deformation is measured.

The analysis assumes that the structural response under the prescribed thermal loading is only weakly affected by the applied compensation. In the present benchmark, the maximum compensation magnitude is below 4% of the plate span, so geometric nonlinearity remains limited and the overall response is expected to change only slightly.

After compensation, the modified geometry is reimported into the finite-element solver and remeshed before analysis. This mirrors mesh regeneration in practical compensation workflows and avoids artificial node-to-node correspondence between the original and compensated models.

Without compensation, pronounced out-of-plane warpage is observed. As summarized in Table 3, the global RMS deformation is 4.62 mm, the mid-span slice RMS is 6.23 mm, and the maximum deformation reaches 6.56 mm. The mid-span slice RMS is computed on the plate mid-span section.

Table 3 Closed-loop effectiveness for the plate benchmark

Metric (out-of-plane, mm)	Uncompensated	DI	Modal
Global RMS	4.62	0.014	0.0061
Mid-span slice RMS	6.23	0.018	0.0085
Global max	6.56	0.095	0.0086

Note: Bold values indicate the lowest error for each reported metric.

After compensation, both direct inversion and the modal formulation reduce the residual deformation by more than two orders of magnitude in the controlled finite-element setting.

As shown in Table 3, the modal formulation gives lower RMS and peak residual values than direct inversion. The remaining deformation field is also smoother and more spatially coherent, as illustrated in Fig. 8.

Under the same loading conditions, both approaches largely remove the benchmark warpage. The modal formulation leaves a smaller and smoother residual field, consistent with restricting the correction to the dominant structural modes.

4.2 General Geometry: L-Bracket

4.2.1 Closed-Loop Validation. The evaluation is next extended to an L-shaped bracket with multiple structural features, including two perpendicular plates, smooth transitions, and internal cutouts. Compared with the plate benchmark, the L-bracket produces a more distributed and coupled warpage response, providing a less compact deformation setting for validation. A reference deformation field is generated for the bracket using the same equivalent thermal-loading framework as in the plate benchmark. The resulting displacement magnitude ranges from approximately 0.05 mm to 2.6 mm.

The bracket deformation combines bending, torsion, and warpage, leading to a broader modal distribution than in the plate benchmark. As a result, more modes are required to reach the same cumulative energy threshold, with the retained dimension increasing to 21 under the same truncation criterion. The complete compensation workflow is shown in Fig. 9. Compensation performance is evaluated by applying the computed correction fields and measuring the closed-loop residual deformation, summarized in Table 4.

Both compensation strategies reduce the global RMS residual relative to the uncompensated case (1.12 mm). Direct inversion lowers the RMS to 1.04 mm, while the modal formulation further reduces it to 1.01 mm.

A clearer difference appears in the peak residual. Direct inversion increases the maximum residual to 2.94 mm, exceeding the

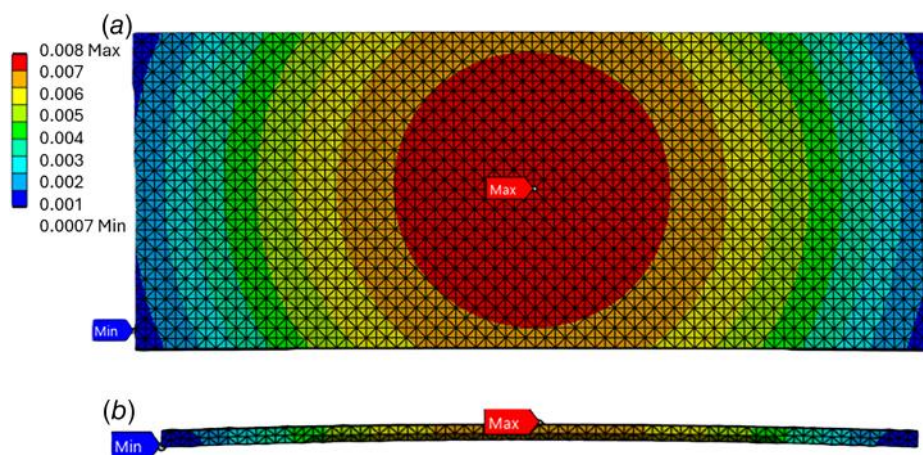


Fig. 8 Final closed-loop deformation of the plate benchmark after single-pass modal compensation, shown in (a) top view and (b) side view

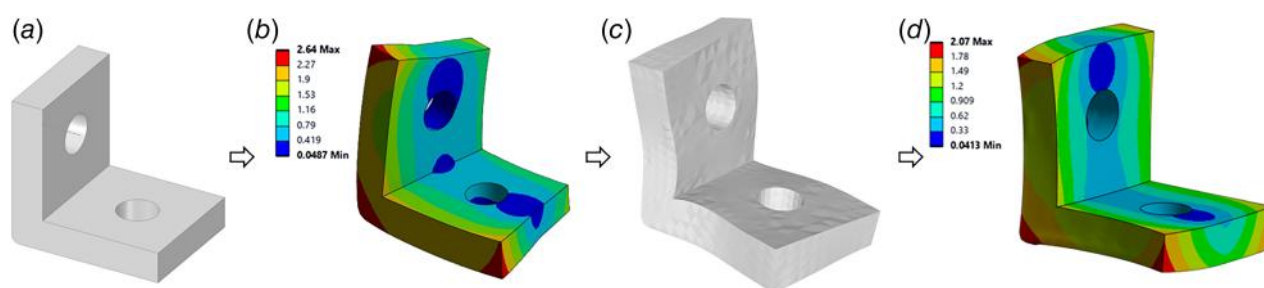


Fig. 9 Closed-loop compensation workflow for the L-bracket: (a) nominal geometry, (b) reference deformation field, (c) modal-based compensated geometry, and (d) closed-loop residual deformation

Table 4 Closed-loop effectiveness for the L-bracket

Method	Global RMS (mm)	Global max (mm)
Uncompensated	1.12	2.64
Direct inversion	1.04	2.94
Modal (proposed)	1.01	2.07

Note: Bold values indicate the lowest error for each reported metric.

uncompensated case (2.64 mm), whereas the modal formulation reduces it to 2.07 mm.

These results suggest that the modal representation provides comparable global correction while avoiding the localized extremes observed under direct inversion. A possible explanation is that full-field inversion retains localized components that contribute little to global RMS reduction but can worsen peak residual after reloading. By restricting the correction to dominant structural modes, the modal formulation yields a smoother response with lower maximum residual.

The improvement remains consistent across both benchmarks, while the reduced margin relative to the plate case is expected for a more spatially distributed response, where the deformation is less concentrated in the dominant structural patterns.

4.2.2 Robustness Under Scan Reconstruction. To examine the effect of realistic measurement and correspondence errors, a scan-reconstruction pipeline is constructed for the L-shaped bracket. Surface points are sampled from the simulated deformation field, followed by rigid misalignment, measurement noise, iterative

closest point (ICP) registration, and nearest-neighbor reconstruction onto the nominal geometry. The resulting observations contain sampling sparsity, measurement noise, and correspondence errors.

The reconstructed field differs substantially from the reference deformation in the geometric domain. It exhibits a relative deviation of approximately 82% in the mass-weighted norm, together with a nodal RMS displacement error of approximately 0.92 mm and an ICP residual RMS of approximately 0.66 mm.

As shown in Fig. 10, the cumulative captured deformation energy curves of the reference and scan-perturbed fields remain close across the modal spectrum, and the retained dimension required to reach the threshold remains unchanged. Despite the geometric discrepancy, the dominant structure-aligned deformation content remains largely preserved in modal coordinates.

Figure 11 shows the resulting compensation fields. Direct inversion follows the perturbed observations and introduces noticeable local protrusions and depressions. In contrast, the modal formulation suppresses these localized artifacts and preserves the main structural correction pattern, yielding a smoother field that remains closer to the unperturbed case.

The visual difference is supported by the mesh-based roughness analysis in Table 5. Direct inversion exhibits larger local angular variation, particularly in the upper tail of the distribution, indicating stronger localized geometric irregularity. The modal compensation remains closer to the correction observed under clean conditions while avoiding sharp local artifacts.

4.3 Mechanism of Modal Stability Under Localized Perturbations. A controlled perturbation study is performed to examine how localized geometric disturbances affect the modal

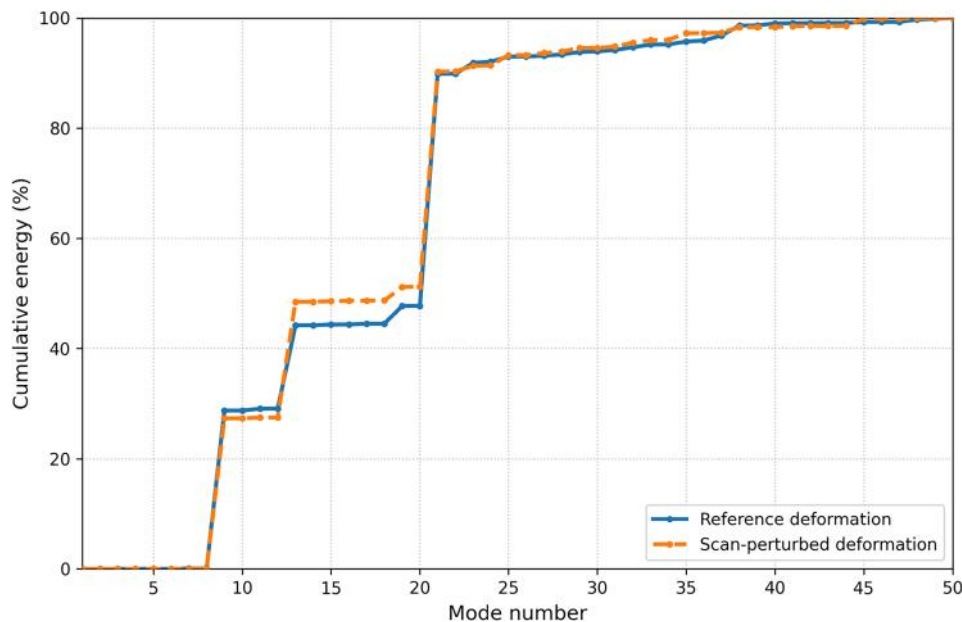


Fig. 10 Cumulative deformation energy captured by the modal basis for the reference and scan-perturbed fields of the L-bracket

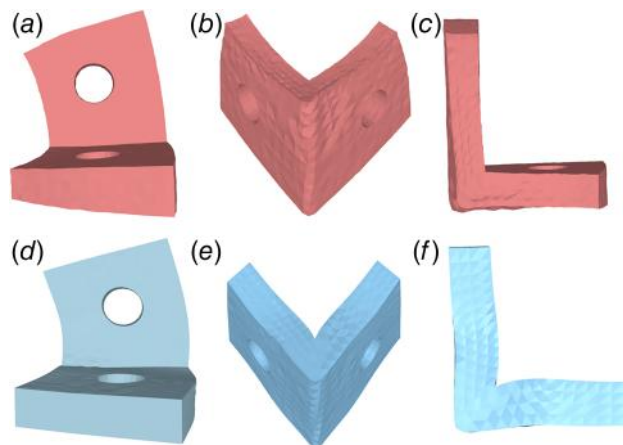


Fig. 11 Comparison of compensated L-bracket geometries from direct inversion and the proposed modal formulation. (a–c) Direct inversion and (d–f) modal compensation, with front, corner, and side views shown from left to right.

Table 5 Compensation output roughness metrics for scan-perturbed compensation fields of the L-bracket

Metric	Direct inversion	Modal
Mean dihedral angle (deg)	9.34	7.77
95th percentile (deg)	39.62	26.77

Note: Bold values indicate the lowest error for each reported metric.

representation. The added disturbances are designed to differ from the dominant structural deformation pattern.

The disturbances are constructed as a superposition of Gaussian bump displacement fields with randomly selected centers and limited spatial support. The perturbation is applied in the out-of-plane direction and scaled to a prescribed RMS displacement magnitude. Because the dominant response is carried by

Table 6 Top-3 modal energy concentration (%) and modal coefficient cosine similarity under localized perturbations with perturbation RMS (mm) for the L-bracket

Perturbation RMS	Top 3 modal energy (%)	Coefficient similarity
0.00	85.86	1.000
0.05	85.82	0.999
0.10	85.64	0.998
0.20	84.91	0.993

smooth global modes, these localized bumps have limited overlap with the primary deformation pattern.

Table 6 reports the ensemble means over 100 random realizations for each perturbation level. The largest perturbation level (0.20 mm RMS) is approximately 10% of the nominal deformation magnitude. As the perturbation RMS increases from 0 to 0.20 mm, the fraction of captured deformation energy in the three dominant modes decreases only slightly, from 85.86% to 84.91%. At the same time, the cosine similarity between the perturbed and reference modal coefficient vectors remains above 0.99 throughout the tested range. These two indicators show that the dominant modal structure changes very little under localized perturbation.

The behavior follows from the weak alignment of the perturbation with the dominant structural modes. The leading modes describe smooth global deformation patterns, whereas the added disturbances are localized and contribute little to the low-order modal subspace. Consequently, even large local geometric errors may produce only limited changes in the dominant modal content. Modal projection therefore retains the global structure-aligned deformation while suppressing localized nonstructural components.

5 Discussion

The results show that the modal formulation works best when the observed deformation is dominated by a small number of structural patterns. This section discusses how this condition relates to additive-manufacturing warpage, practical compensation workflows, and the limits of the proposed method.

5.1 Process Relevance and Application Workflow. In additive manufacturing, warpage is a common process-induced deformation that often develops from nonuniform heating and cooling during layerwise processing. Thermal gradients and repeated thermal cycles generate residual stresses, and constraint release from supports or the build platform can further change the final shape. Although these mechanisms are process driven, the resulting distortion is not determined by the process history alone. It is also shaped by the geometry, spngness distribution, and boundary conditions of the part. For this reason, the observed deformation often appears as global bending, twisting, wall deflection, or plate-like warpage rather than as independent local offsets.

This observation motivates the proposed compensation strategy. The method does not reconstruct the full thermo-mechanical history of the build. Instead, it starts from a deformation field obtained from simulation or measurement and represents that field in a structural modal basis. When the deformation is dominated by a few structural patterns, modal projection preserves the main warpage response while reducing local measurement, registration, and correspondence artifacts before inversion.

The framework is therefore most relevant to AM cases where the dominant dimensional error is a coherent structural distortion. Although the underlying process mechanisms differ substantially across AM processes, many final geometric errors are still shaped by the part geometry, spngness distribution, and boundary conditions. Examples include residual-stress-induced warpage in metal powder-bed fusion, thin-wall deflection in directed energy deposition, and thermally induced bending in polymer material extrusion. In a practical workflow, the nominal geometry is first used to compute the structural modes. The simulated or measured as-built geometry is then registered and brought into correspondence with the nominal geometry to obtain the observed deformation field. This field is projected onto the retained modes, and the projected deformation is inverted to generate a predeformed design. In this way, the method can be added as a projection step before conventional inversion without replacing the overall compensation workflow.

5.2 Boundary and Applicability Analysis. The above workflow is effective when the final deformation remains compact in the nominal modal basis. Although the structural modes are derived from the linearized nominal geometry, their use does not require the manufacturing process itself to be strictly linear. Nonlinear process effects may change the magnitude, asymmetry, or coupling of the deformation, but the leading modes can still be useful if the primary shape change remains governed by a small number of structural patterns. In such cases, the dominant modes capture the main deformation, while a limited number of additional modes represent secondary components.

However, a practical limit is reached when the deformation is no longer compact in the nominal modal basis, regardless of the specific manufacturing source. This may occur under strong distributed warpage, distortion near support-contact regions, localized yielding, pronounced local curvature, or changing support conditions during build or removal. As the response spreads across more modes, the benefit of modal reduction decreases. If the full modal basis is retained, the formulation becomes equivalent to the original full displacement representation and reduces to full-field inversion.

Beyond these representation limits, measurement and correspondence errors introduce a separate challenge. The preceding experiments show that moderate perturbations can often be tolerated. A more difficult case is coherent registration or reconstruction bias, such as a slightly tilted scanned surface, a consistent shift on one side, or a broad region reconstructed too high or too low. Because these errors can resemble true low-order bending or twisting patterns, they may introduce systematic bias into the compensation field.

5.3 Future Work. These limitations suggest two natural extensions of the current framework.

Nonlinear structure-aligned representation. When geometric or material nonlinearities create strong mode interactions, a linear modal representation becomes insufficient. A natural next step is to extend the current subspace to a nonlinear structure-aligned representation, for example through modal derivatives as used in quadratic modal formulations. Such an extension would retain the compactness of the present approach while capturing coupled deformation patterns beyond linear superposition. It would also provide a natural framework for studying sequential or iterative compensation updates under nonlinear conditions, including their convergence and stability across repeated correction cycles.

Unified representation for deformation and correspondence. A second direction is to use the same modal representation not only for compensation but also for geometric registration. Instead of assigning correspondence through pointwise matching, the nominal mesh can be deformed within the structure-aligned modal space to fit the observed shape. In this setting, correspondence is induced by the fitted deformation itself rather than introduced as a separate preprocessing step. This idea is being investigated in parallel work by the present authors, where deformation between as-designed and as-built geometries is represented through a superposition of structural mode shapes. It points toward a more unified compensation framework, in which deformation representation, correspondence estimation, and compensation are handled within the same structure-aligned space.

6 Conclusion

Geometric compensation is fundamentally a problem of representation rather than inversion. When deformation is governed by structural response, direct inversion in the full displacement space can transfer noise, correspondence errors, and local artifacts into the compensation field.

This work establishes a structure-aligned modal compensation formulation, where compensation is constructed in a reduced modal subspace and inversion is applied only after the observed deformation has been restricted to dominant structural deformation patterns. In the limiting case where all shape-changing modes are retained, the formulation reduces to direct inversion, showing that the improvement comes from the chosen representation space rather than from modifying the inversion step.

Finite-element numerical studies show that, in the tested cases, the proposed formulation produces smoother compensation fields, lower closed-loop residual deformation, and improved robustness to measurement and correspondence perturbations compared with direct inversion. In practice, the method can be implemented as a projection step prior to inversion, allowing straightforward integration into existing additive manufacturing compensation workflows. More broadly, this work provides a physically interpretable representation framework for robust geometric compensation under imperfect deformation observations.

Conflict of Interest

There are no conflicts of interest.

Data Availability Statement

The datasets generated and supporting the findings of this article are obtainable from the corresponding author upon reasonable request.

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